Fluid Properties

- The impulse-momentum equation is one of the basic tools (other being continuity and Bernoulli's equations) for the solution of flow problems. Its application leads to the solution of problems in fluid mechanics which cannot be solved by energy principles alone. Sometimes it is used in conjunction with the energy equation to obtain complete solution of engineering problems. The momentum equation is based on the law of conservation of momentum or momentum principle which states as follows: "The net force acting on a mass of fluid is equal to change in momentum of flow per unit time in that direction".
- As per Newton's second law of motion,
- F = ma, where, m = Mass of fluid,
- F = Force acting on the fluid, and
- a = Acceleration (acting in the same direction as F).

But acceleration, $a = \frac{dv}{dt}$

$\therefore$ $F = m.\frac{dv}{dt} = \frac{d(mv)}{dt}$

- ('m' is taken inside the differential, being constant)
- This equation is known as momentum principle. It can also be written as:
- F.dt = d (mv) ...(6.19)
- This equation is known as Impulse-momentum equation. It may be stated as follows: "The impulse of a force F acting on a fluid mass 'm' in a short interval of time dt is equal to the change of momentum d(mv) in direction of force".
- The impulse-momentum equations are often called simply momentum equations.
- Applications of impulse-momentum equation:
- The impulse-momentum equation is used in the following types of problems:
- 1. To determine the resultant force acting on the boundary of flow passage by a stream of fluid as the stream changes its direction, magnitude or both. Problems of this type are: (i) Pipe bends, (ii) Reducers, (iii) Moving vanes, (iv) Jet propulsion, etc.
- 2. To determine the characteristic of flow when there is an abrupt change of flow section. Problems of this type are:
- (i) Sudden enlargement in a pipe, (ii) Hydraulic jump in a channel, etc.

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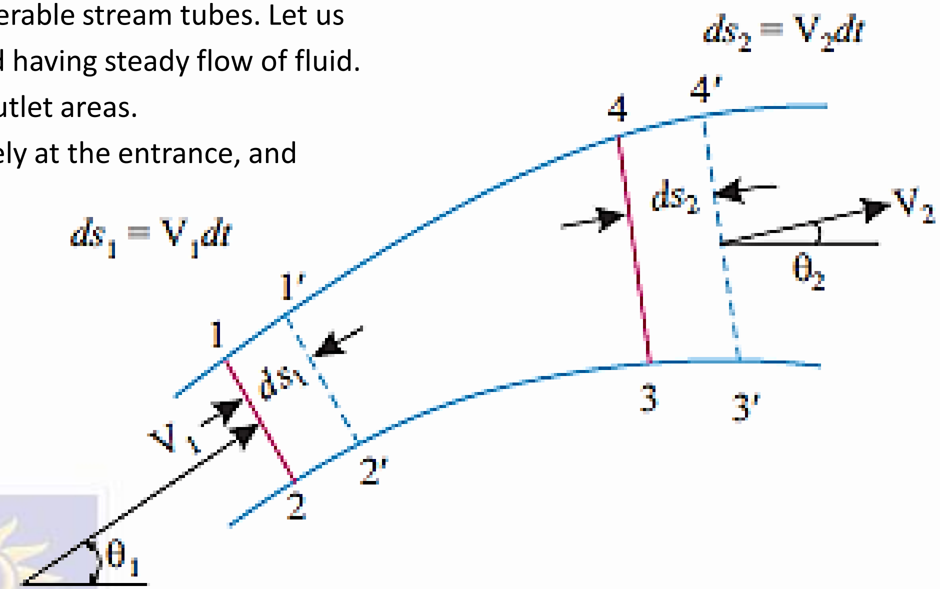
STEADY FLOW MOMENTUM EQUATION:



- The entire flow space may be considered to be made up of innumerable stream tubes. Let us
- consider one such stream tube lying in the X-Y plane (Fig 6.48) and having steady flow of fluid.
- Flow can be assumed to be uniform and normal to the inlet and outlet areas.
- Let, V1,ρ1 = Average velocity and density (of fluid mass) respectively at the entrance, and
- V2, ρ2 = Average velocity and density respectively at the exit.
- Further let the mass of fluid in the region
- 1 2 3 4 shifts to new position 1′ 2′ 3′ 4′
- due to the effect of external forces on the
- stream after a short interval. Due to gradual
- increase in the flow area in the direction of
- flow, velocity of fluid mass and hence the
- momentum is gradually reduced. Since the
- area 1′ 2′ 3 4 is common to both the regions
- 1 2 3 4 and 1′ 2′ 3′ 4′, therefore, it will not
- experience any change in momentum. Obviously, then the changes in momentum of the fluid masses in the sections 1 2 2′ 1′
- and 4 3 3′ 4′ will have to be considered. According to the principle of mass conservation, Fluid mass with the region 1 2 2′ 1′ = Fluid mass within the region 4 3 3′ 4′ ρ1 A1 ds1 = ρ2A2ds2 ...(6.20) ∴ Momentum of fluid mass contained in the region 1 2 2′ 1′
- = (ρ1A1 ds1) V1 = (ρ1A1 V1. dt) V1
- Momentum of fluid mass contained in the region 4 3 3′ 4′
- = (ρ2A2 ds2) V2 = (ρ2A2 V2. dt) V2
- ∴ Change in momentum = (ρ2A2 V2.dt) V2 – (ρ1A1 V1. dt) V1 But, ρ1 = ρ2 = ρ ...for steady incompressible flow
- and, A1V1 = A2V2 = Q ...from continuity considerations ∴ Change in momentum = ρQ(V2 – V1) dt
- Using impulse-momentum principle, we have:
- Fdt = ρQ (V2 – V1)dtv

$$F = \frac{wQ}{g}(V_2 - V_1)$$

Resolving V_1 and V_2 along X-axis and Y-axis, we get:

Components along *X-axis*: $V_1 \cos\theta_1$ and $V_2 \cos\theta_2$

Components along Y- axis : $V_1 \sin\theta_1$ and $V_2 \sin\theta_2$

(where, θ_1 and θ_2 are the inclinations with the horizontal of the centre line of the pipe at 1-2 and 3-4).

$\therefore$ Components of force F along X-axis and Y-axis are:

$$F_x = \frac{wQ}{g}(V_2\cos\theta_2 - V_1\cos\theta_1)$$

$$F_y = \frac{WQ}{g}(V_2\sin\theta_2 - V_1\sin\theta_1) \qquad ...(6.23)$$

Eqn. (6.23) represents the components of the force exerted by the *pipe bend on the fluid mass*. Usually, we are interested in the forces by the fluid on the pipe bend. Since action and reaction are equal and opposite (Newton's third law of motion), the fluid mass would exert the same force on the pipe bend but in *opposite direction* and as such the force components exerted by the fluid on the pipe bend are given as follows:

$$\left.\begin{aligned} F_x &= \frac{wQ}{g}(V_1\cos\theta_1 - V_2\cos\theta_2) \\ F_y &= \frac{wQ}{g}(V_1\sin\theta_1 - V_2\sin\theta_2) \end{aligned}\right\} \qquad ...(6.24)$$

Since the *dynamic* forces (eqn. 6.23) must be supplemented by the *static pressure forces* acting over the inlet and outlet sections, therefore, we have:

$$\left.\begin{aligned} F_x &= \frac{wQ}{g}(V_1\cos\theta_1 - V_2\cos\theta_2) + p_1A_1\cos\theta_1 - p_2A_2\cos\theta_2 \\ F_y &= \frac{wQ}{g}(V_1\sin\theta_1 - V_2\sin\theta_2) + p_1A_1\sin\theta_1 - p_2A_2\sin\theta_2 \end{aligned}\right\} \qquad ...(6.25)$$

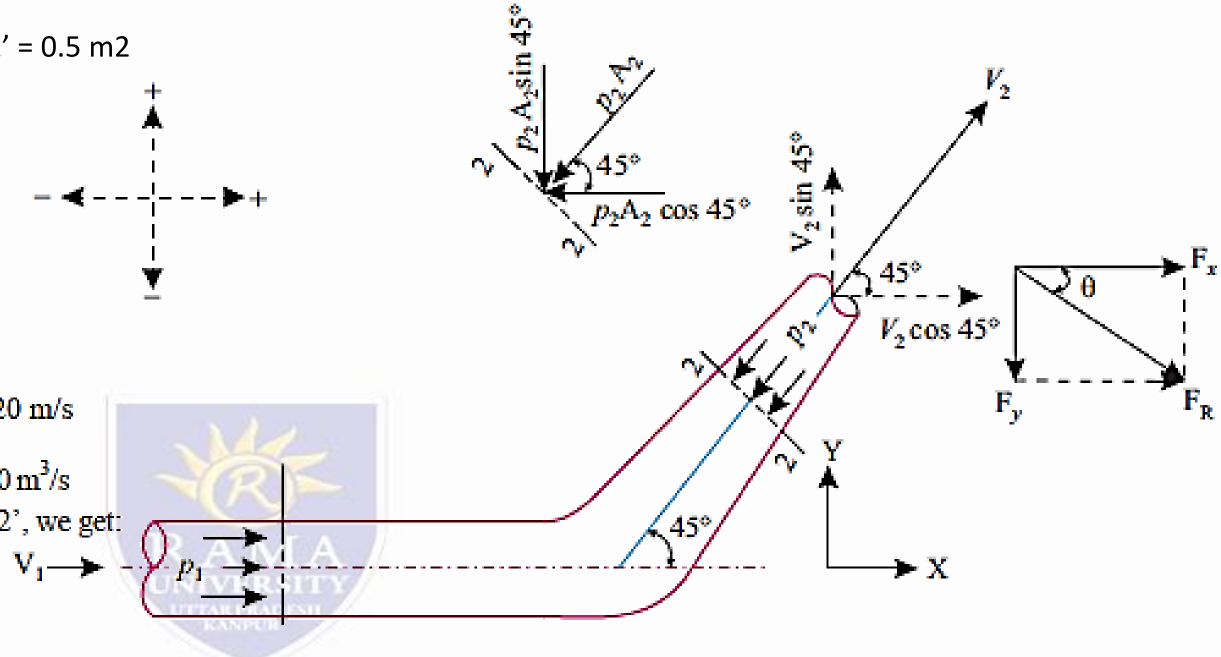
The magnitude of the resultant force acting on the pipe bend,

$$F_R = \sqrt{F_x^2 + F_y^2} \qquad ...(6.26)$$

and, the direction of the resultant force with X-axis,

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) \qquad ...[6.26\,(a)]$$

- In a 45° bend a rectangular air duct of 1 m2 cross-sectional area is gradually reduced to 0.5 m2 area. Find the magnitude and direction of force required to hold the duct in position if the velocity of flow at 1 m2 section is 10 m/s, and pressure is 30 kN/m2. Take the specific weight of air as 0.0116 kN/m3
- Area at section, ‘1’ = 1 m2; Area at section ‘2’ = 0.5 m2
- Velocity at section ‘1’, V1 = 10 m/s
- Pressure at section ‘1’, p1 = 30 kN/m2
- Sp. weight of air, w = 0.0116 kN/m3
- As per continuity equation,



As per continuity equation,

$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{1 \times 10}{0.5} = 20 \text{ m/s}$$

$$\text{Discharge, } Q = A_1V_1 = 1 \times 10 = 10 \text{ m}^3/\text{s}$$

Applying Bernoulli's equation at sections ‘1’ and ‘2’, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

But, $z_1 = z_2$

$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g}$$

$$\frac{30}{0.0116} + \frac{10^2}{2 \times 9.81} = \frac{p_2}{w} + \frac{20^2}{2 \times 9.81}$$

$$2586 + 5.1 = \frac{p_2}{w} + 20.4$$

$$\frac{p_2}{w} = 2586 + 5.1 - 20.4 = 2570.7$$

or, $p_2 = 2570.7 \times 0.0116 = 29.82 \text{ kN/m}^2$

Magnitude and direction of force (resultant) F_R:

Force along X-axis:

$$F_x = \frac{wQ}{g}(V_{1x} - V_{2x}) + (p_1A_1)_x + (p_2A_2)_x$$

where, $V_{1x} = 10 \text{ m/s};\ V_{2x} = V_2 \cos 45° = 20 \times 0.707 = 14.14 \text{ m/s}$

$(p_1A_1)_x = p_1A_1 = 30 \times 1 = 30 \text{ kN};\ (p_2A_2)_x = -p_2A_2 \cos 45° = -29.82 \times 0.5 \times 0.707 = -10.54 \text{ kN}$

$$\therefore \quad F_x = \frac{0.0116}{9.81} \times 10\,(10 - 14.14) + 30 - 10.54 = 19.41 \text{ kN } (\rightarrow)$$

Force along Y-axis:

$$F_y = \frac{wQ}{g}(V_{1y} - V_{2y}) + (p_1A_1)_y + (p_2A_2)_y$$

where, $V_{1y} = 0;\ V_{2y} = V_2 \sin 45° = 20 \times 0.707 = 14.14 \text{ m/s}$

$(p_1A_1)_y = 0;\ (p_2A_2)_y = -p_2A_2 \sin 45° = -29.82 \times 0.5 \times 0.707 = -10.54 \text{ kN}$

$$\therefore \quad F_y = \frac{0.0116 \times 10}{9.81}(0 - 14.14) + 0 - 10.54 = -10.71 \text{ kN } (\downarrow)$$

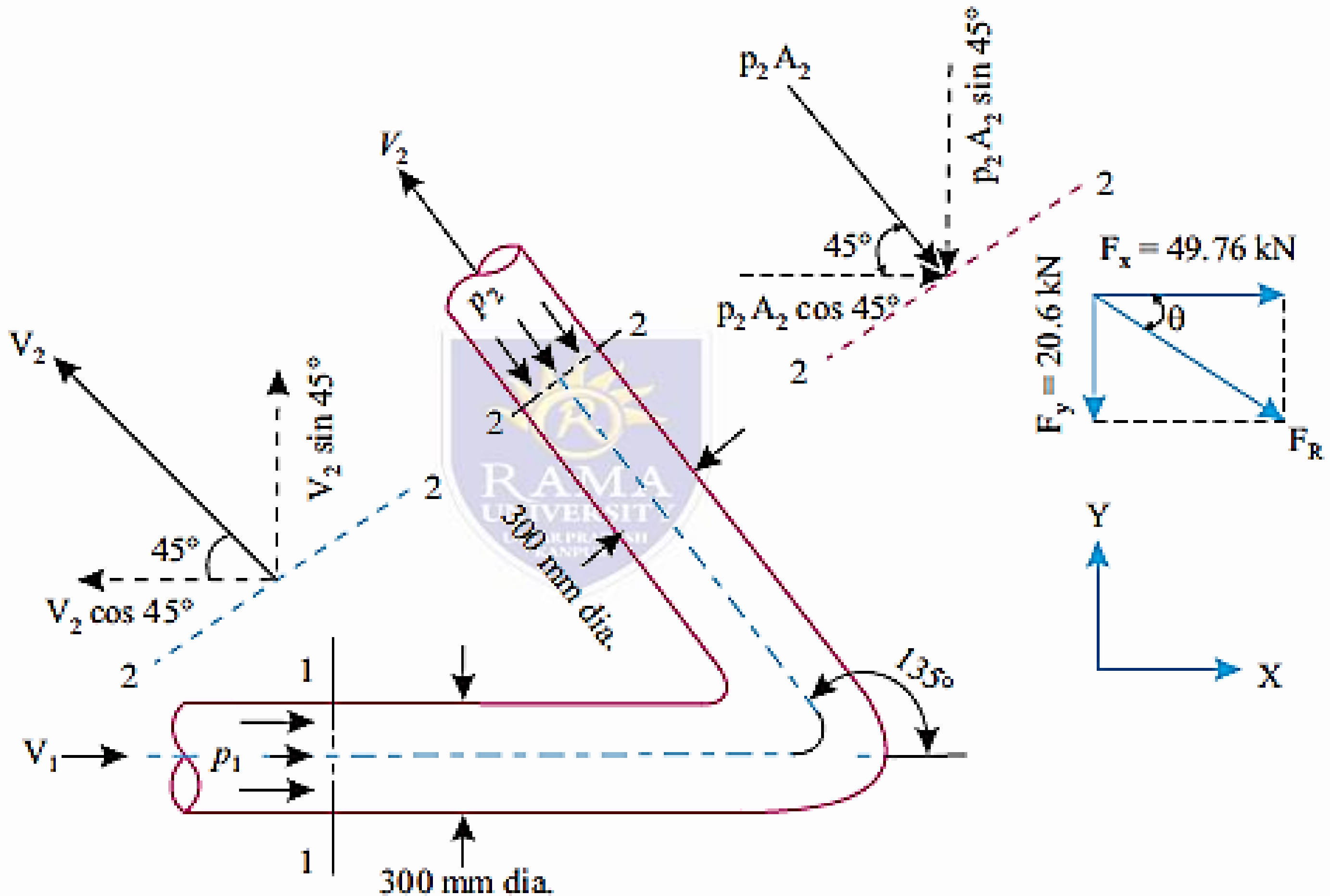
$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(19.41)^2 + (10.71)^2} = 22.17 \text{ kN}$ **(Ans.)**

The direction of F_R with X-axis is given as:

$$\tan\theta = \frac{F_y}{F_x} = \frac{10.71}{19.41} = 0.5518$$

or, $\theta = \tan^{-1} 0.5518 = 28.88°$ or $28°53'$ **(Ans.)**

- 250 litres/sec. of water is flowing in a pipe having a diameter of 300 mm. If the pipe is bent by 135°, find the magnitude and direction of the resultant force on the bend. The pressure of the water flowing is 400 kN/m2. Take specific weight of water as 9.81 kN/m3.



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