Example *The work cycle of a mechanical component subjected to completely reversed bending stresses consists of the following three elements:*

(i) ± 350 N/mm² for 85% of time
(ii) ± 400 N/mm² for 12% of time
(iii) ± 500 N/mm² for 3% of time

The material for the component is 50C4 (S_{ut} = 660 N/mm²) and the corrected endurance limit of the component is 280 N/mm². Determine the life of the component.

Solution

Given $S_{ut} = 660$ N/mm² $S_e = 280$ N/mm²

Step I *Construction of S–N diagram*

$0.9S_{ut} = 0.9(660) = 594$ N/mm²

$\log 10\ (0.9S_{ut}) = \log_{10}(594) = 2.7738$

$\log_{10}(S_e) = \log_{10}(280) = 2.4472$

$\log_{10}(\sigma_1) = \log_{10}(350) = 2.5441$

$\log_{10}(\sigma_2) = \log_{10}(400) = 2.6021$

$\log_{10}(\sigma_3) = \log_{10}(500) = 2.6990$

The *S–N* curve for this problem is shown in

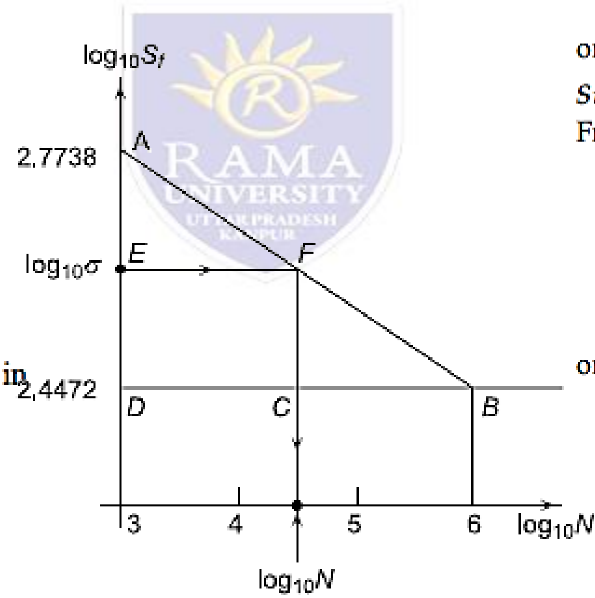


Fig. 5.38

$$\overline{EF} = \frac{\overline{DB} \times \overline{AE}}{\overline{AD}} = \frac{(6-3)(2.7738 - \log_{10}\sigma)}{(2.7738 - 2.4472)} \quad \text{(a)}$$

and $\log_{10} N = 3 + \overline{EF}$ (b)

From (a) and (b),

$\log_{10} N = 3 + 9.1855\ (2.7738 - \log_{10}\sigma)$

Therefore,

$\log_{10}(N_1) = 3 + 9.1855\ (2.7738 - 2.5441)$

or $N_1 = 128\,798$

$\log_{10}(N_2) = 3 + 9.1855\ (2.7738 - 2.6021)$

or $N_2 = 37\,770$

$\log_{10}(N_3) = 3 + 9.1855\ (2.7738 - 2.6990)$

or $N_3 = 4865$

Step III *Fatigue life of component*

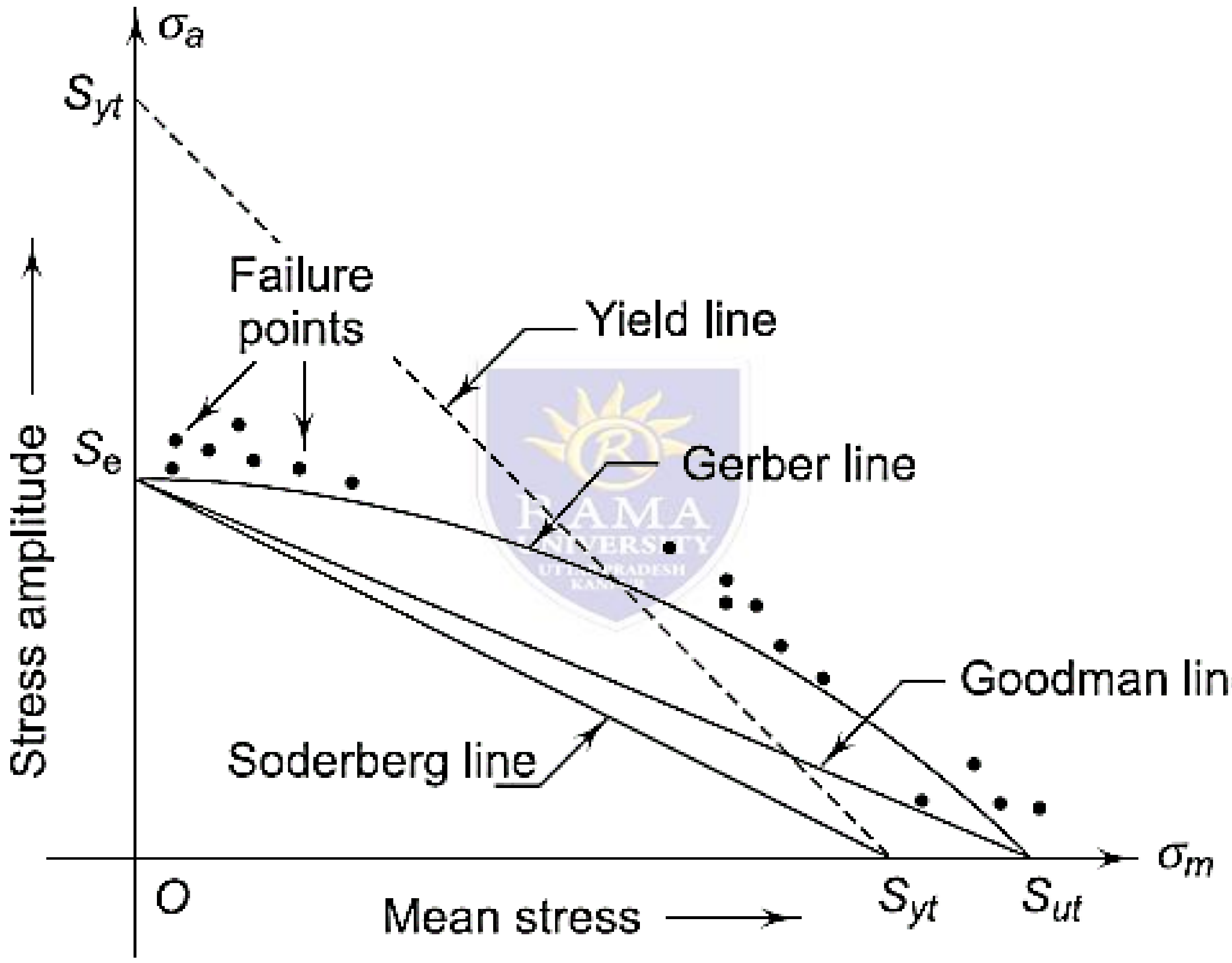
From Eq. 5.34,

$$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \frac{\alpha_3}{N_3} = \frac{1}{N}$$

$$\frac{0.85}{128\,798} + \frac{0.12}{37\,770} + \frac{0.03}{4865} = \frac{1}{N}$$

or $N = 62\,723$ cycles

- **SODERBERG AND GOODMAN LINES**



- Gerber Line A parabolic curve joining Se on the ordinate to Sut on the abscissa is called the Gerber line.
- Soderberg Line A straight line joining Se on the ordinate to Syt on the abscissa is called the Soderberg line.
- Goodman Line A straight line joining Se on the ordinate to Sut on the abscissa is called the Goodman line.

We will apply following form for the equation of a straight line,

$$\frac{x}{a} + \frac{y}{b} = 1$$

where a and b are the intercepts of the line on the X and Y axes respectively.

Applying the above formula, the equation of the Soderberg line is given by,

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = 1 \qquad (5.36)$$

Similarly, the equation of the Goodman line is given by,

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = 1 \qquad (5.37)$$

The Goodman line is widely used as the criterion of fatigue failure when the component is subjected to mean stress as well as stress amplitude. It is because of the following reasons:

(i) The Goodman line is safe from design considerations because it is completely inside the failure points of test data.

(ii) The equation of a straight line is simple compared with the equation of a parabolic curve.

(iii) It is not necessary to construct a scale diagram and a rough sketch is enough to construct fatigue diagram.

- MODIFIED GOODMAN DIAGRAMS

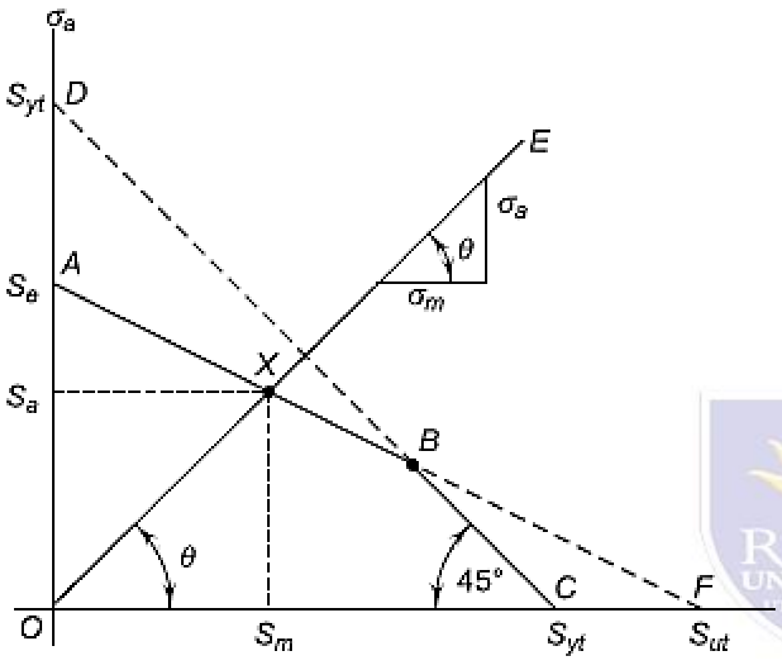


Fig. 5.40 *Modified Goodman Diagram for Axial and Bending Stresses*

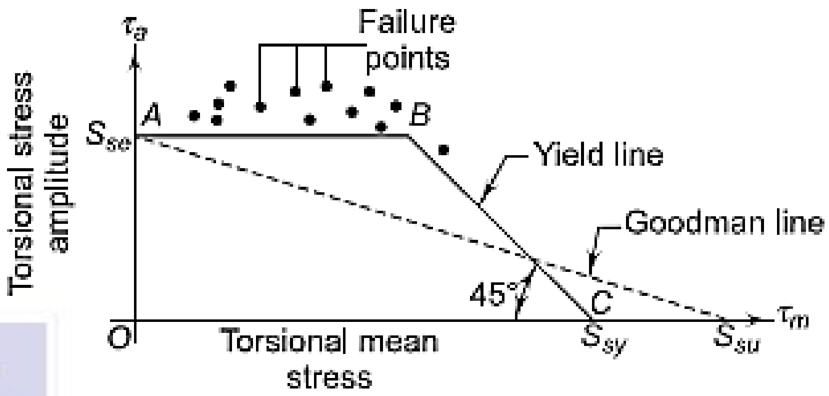


Fig. 5.41 *Modified Goodman Diagram for Torsional Shear Stresses*

- The components, which are subjected to
- fl uctuating stresses, are designed by constructing
- the modifi ed Goodman diagram. For the purpose of
- design, the problems are classifi ed into two groups:
- (i) components subjected to fl uctuating axial or
- bending stresses; and
- (ii) components subjected to fl uctuating torsional
- shear stresses. While solving a problem, a line OE with a slope
- of tan q is constructed in such a way that,

$$\tan\theta = \frac{\sigma_a}{\sigma_m} \qquad (5.38)$$

Since $$\frac{\sigma_a}{\sigma_m} = \frac{(P_a / A)}{(P_m / A)} = \frac{P_a}{P_m}$$

$$\therefore \qquad \tan\theta = \frac{P_a}{P_m} \qquad (5.39)$$

The magnitudes of P_a and P_m can be determined from maximum and minimum forces acting on the component.

Similarly, it can be proved that

$$\tan\theta = \frac{(M_b)_a}{(M_b)_m} \qquad (5.40)$$

The magnitudes of $(M_b)_a$ and $(M_b)_m$ can be determined from maximum and minimum bending moment acting on the component.

The point of intersection of lines AB and OE is X. The point X indicates the dividing line between the safe region and the region of failure. The coordinates of the point X (Sm, Sa) represent the limiting values of stresses, which are used to calculate the dimensions of the component. The permissible stresses are as follows:

$$\sigma_a = \frac{S_a}{(fs)} \quad \text{and} \quad \sigma_m = \frac{S_m}{(fs)} \qquad (5.41)$$

The modifi ed Goodman diagram for fl uctuating torsional shear stresses is shown in Fig. 5.41. In this diagram, the torsional mean stress is plotted on theabscissa while the torsional stress amplitude on the ordinate. The torsional yield strength Ssy is plotted on the abscissa and the yield line is constructed, whichis inclined at 45° to the abscissa. The point of intersection of this line and the yield line is B. The area OABC represents the region of safety in this case. It is not necessary to construct a fatigue diagram for fl uctuating torsional shear stresses because AB is parallel to the X-axis. Instead, a fatigue failure is indicated if,

$$\tau_a = S_{se} \qquad (5.42)$$

and a static failure is indicated if,

$$\tau_{max} = \tau_a + \tau_m = S_{sy} \qquad (5.43)$$

The permissible stresses are as follows:

$$\tau_a = \frac{S_{se}}{(fs)} \qquad (5.44)$$

and $$\tau_{max} = \frac{S_{sy}}{(fs)} \qquad (5.45)$$