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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 33

Topic: Strain Energy in various loading conditions

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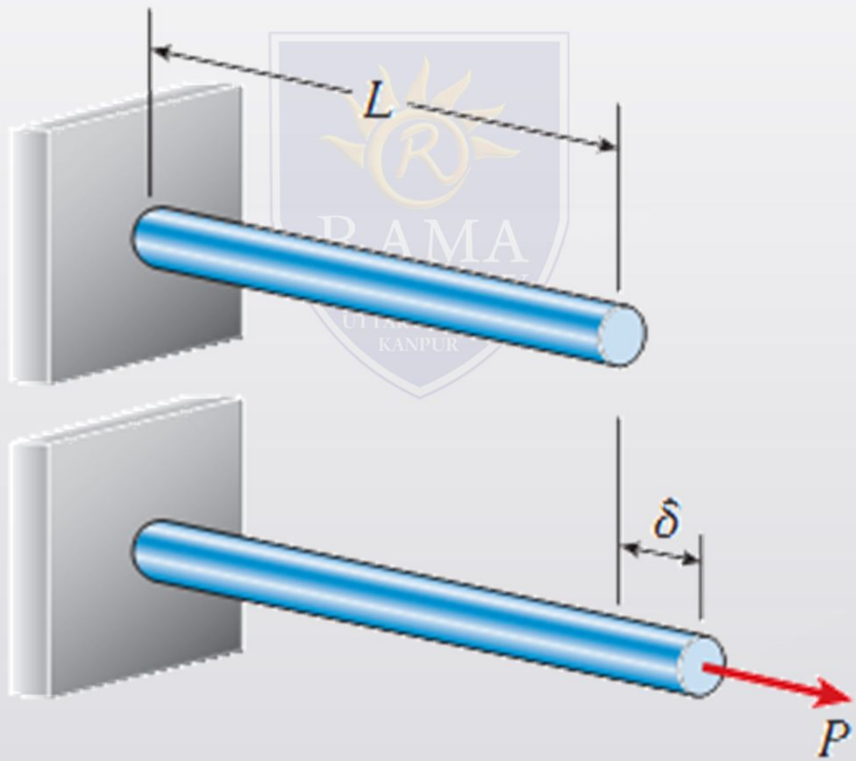
Strain Energy in Axially Loaded members

✓ Strain energy is a fundamental concept in applied mechanics, and strain-energy principles are widely used for determining the response of machines and structures to both static and dynamic loads.

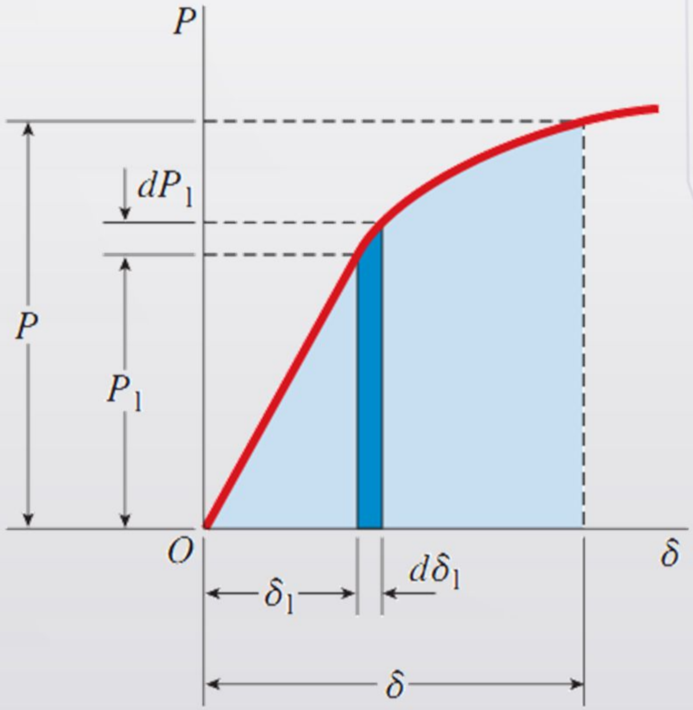


Strain Energy in Axially Loaded members

- ✓ Consider a prismatic bar of length L subjected to a tensile force P
  - ✓ We assume that the load is applied slowly, so that it gradually increases from zero to its maximum value P. Such a load is called a static load because there are no dynamic or inertial effects due to motion.
- ✓ The bar gradually elongates as the load is applied, eventually reaching its maximum elongation δ at the same time that the load reaches its full value P. Thereafter, the load and elongation remain unchanged.
- ✓ During the loading process, the load P moves slowly through the distance δ and does a certain amount of work. To evaluate this work, we recall from elementary mechanics that a constant force does work equal to the product of the force and the distance through which it moves.

Strain Energy in Axially Loaded members

- ✓ In this case the force varies in magnitude from zero to its maxi-mum value P. To find the work done by the load under these conditions, we need to know the manner in which the force varies. This information is supplied by a load-displacement diagram



$$W = \int_0^{\delta} P_1 \, d\delta_1$$

In geometric terms, the work done by the load is equal to the area below the load-displacement curve.

Strain Energy in Axially Loaded members

- When the load stretches the bar, strains are produced. The presence of these strains increases the energy level of the bar itself. Therefore, a new quantity, called strain energy, is defined as the energy absorbed by the bar during the loading process. From the principle of conservation of energy, we know that this strain energy is equal to the work done by the load provided no energy is added or subtracted in the form of heat.

$$U = W = \int_0^{\delta} P_1 d\delta_1$$

- U is the symbol for strain energy. Sometimes strain energy is referred to as internal work to distinguish it from the external work done by the load.
  - The strain energy recovered during unloading, called the elastic strain energy
  - Energy that is lost in the process of permanently deforming the bar. This energy is known as the inelastic strain energy.

Linearly Elastic Behavior

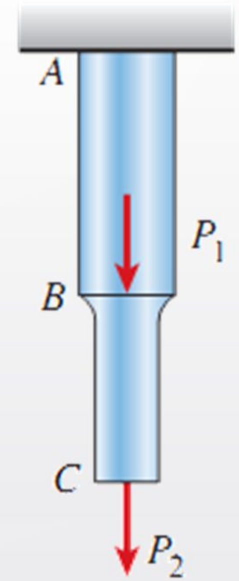
$$U = W = \frac{P\delta}{2}$$

We know that

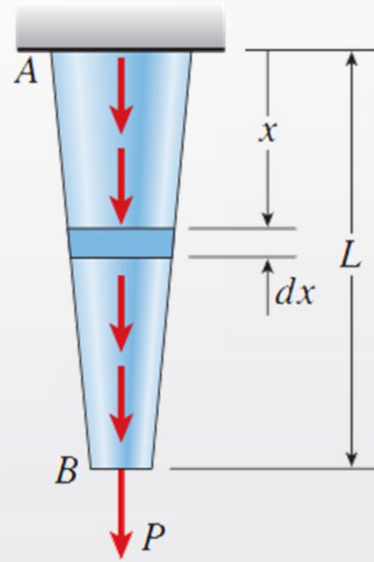
$$\delta = \frac{PL}{EA}$$

Therefore

$$\frac{P^2L}{2EA} \; and \; \frac{EA\delta^2}{2L}$$



$$U = \sum_{i=1}^{n} \frac{P_i^2 L_i}{2E_i A_i}$$



$$U = \int \frac{P^2(x)}{2EA(x)} dx$$

Strain energy is not a linear function of the loads, not even when the material is linearly. Therefore we cannot obtain the strain energy of a structure supporting more than one load by combining the strain energies obtained from the individual loads acting separately.

Strain-Energy Density

- ✓ In many situations it is convenient to use a quantity called strain-energy density, defined as the strain energy per unit volume of material.
- ✓ Strain energy per unit volume of material (details in next lecture)
  - ✓ For material under tensile or compressive stress

$$u = \frac{1}{2}\sigma\epsilon = \frac{\sigma^2}{2E} = \frac{E\epsilon^2}{2}$$

  - ✓ For material under pure shear

$$u = \frac{1}{2}\tau\gamma = \frac{\tau^2}{2G} = \frac{G\gamma^2}{2}$$

  - ✓ Shaft under constant torque

$$Solid\ u = \frac{\tau^2}{4G}, Hollow\ u = \frac{\tau^2}{4G}\left(1 + \frac{d_o^2}{d_i^2}\right)$$

Strain-Energy Density in Pure Shear

- **Resilience**
  - Ability of a material to absorb energy in the elastic region when is it strained
- **Proof resilience**
  - Maximum energy absorbing capacity of a material in the elastic region

$$\frac{1}{2} P_{EL} \times \delta_{EL}$$

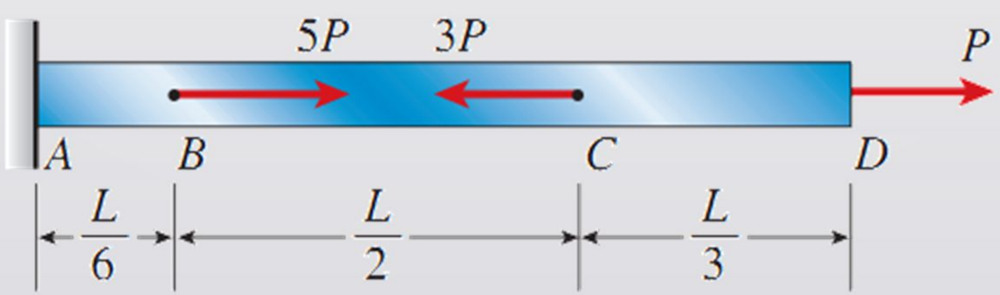
- Another quantity, called *toughness*, refers to the ability of a material to absorb energy without fracturing. The corresponding modulus, called the **modulus of toughness u_t**, is the strain-energy density when the material is stressed to the point of failure.

Question

2.7-1 A prismatic bar *AD* of length *L*, cross-sectional area *A*, and modulus of elasticity *E* is subjected to loads 5*P*, 3*P*, and *P* acting at points *B*, *C*, and *D*, respectively (see figure). Segments *AB*, *BC*, and *CD* have lengths *L*/6, *L*/2, and *L*/3, respectively.

(a) Obtain a formula for the strain energy *U* of the bar.

(b) Calculate the strain energy if $P = 27$ kN, $L = 130$ cm, $A = 18$ cm^2, and the material is aluminum with 72 GPa.

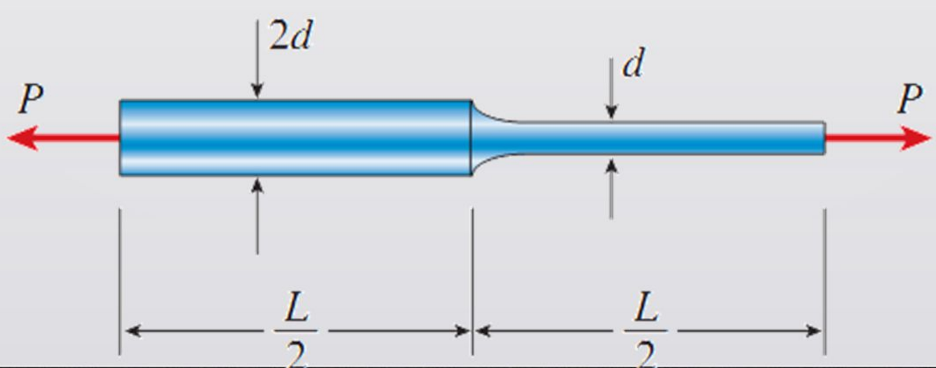


Question

2.7-2 A bar of circular cross section having two different diameters d and $2d$ is shown in the figure. The length of each segment of the bar is $L/2$ and the modulus of elasticity of the material is E.

(a) Obtain a formula for the strain energy U of the bar due to the load P.

(b) Calculate the strain energy if the load $P = 27$ kN, the length $L = 600$ mm, the diameter $d = 40$ mm, and the material is brass with $E = 105$ GPa.



Question

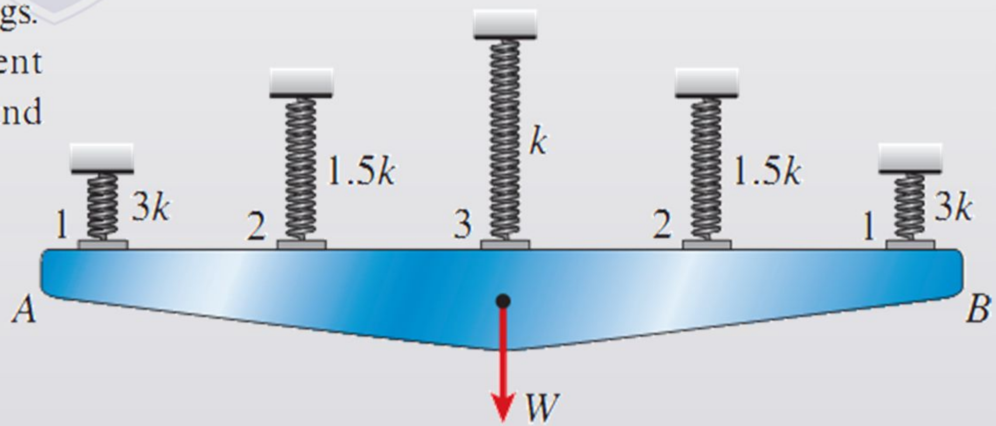
2.7-8 The statically indeterminate structure shown in the figure consists of a horizontal rigid bar AB supported by five equally spaced springs. Springs 1, 2, and 3 have stiffnesses $3k$, $1.5k$, and k, respectively. When unstressed, the lower ends of all five springs lie along a horizontal line. Bar AB, which has weight W, causes the springs to elongate by an amount δ.

(a) Obtain a formula for the total strain energy U of the springs in terms of the downward displacement δ of the bar.

(b) Obtain a formula for the displacement δ by equating the strain energy of the springs to the work done by the weight W.

(c) Determine the forces F_1, F_2, and F_3 in the springs.

(d) Evaluate the strain energy U, the displacement δ, and the forces in the springs if $W = 600$ N and $k = 7.5$ N/mm.



Question

2.7-9 A slightly tapered bar *AB* of rectangular cross section and length *L* is acted upon by a force *P* (see figure). The width of the bar varies uniformly from b_2 at end *A* to b_1 at end *B*. The thickness *t* is constant.

(a) Determine the strain energy *U* of the bar.

(b) Determine the elongation δ of the bar by equating the strain energy to the work done by the force *P*.

Question

2.7-12 A bungee cord that behaves linearly elastically has an unstressed length $L_0 = 760$ mm and a stiffness $k = 140$ N/m. The cord is attached to two pegs, distance $b = 380$ mm apart, and pulled at its midpoint by a force $P = 80$ N (see figure).

(a) How much strain energy U is stored in the cord?

(b) What is the displacement δ_C of the point where the load is applied?

(c) Compare the strain energy U with the quantity $P\delta_C/2$.

(*Note:* The elongation of the cord is *not* small compared to its original length.)