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FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mechanical Engineering

MEPS102:Strength of Material

Lecture 28

Topic: Introduction, Buckling and stability

Instructor:

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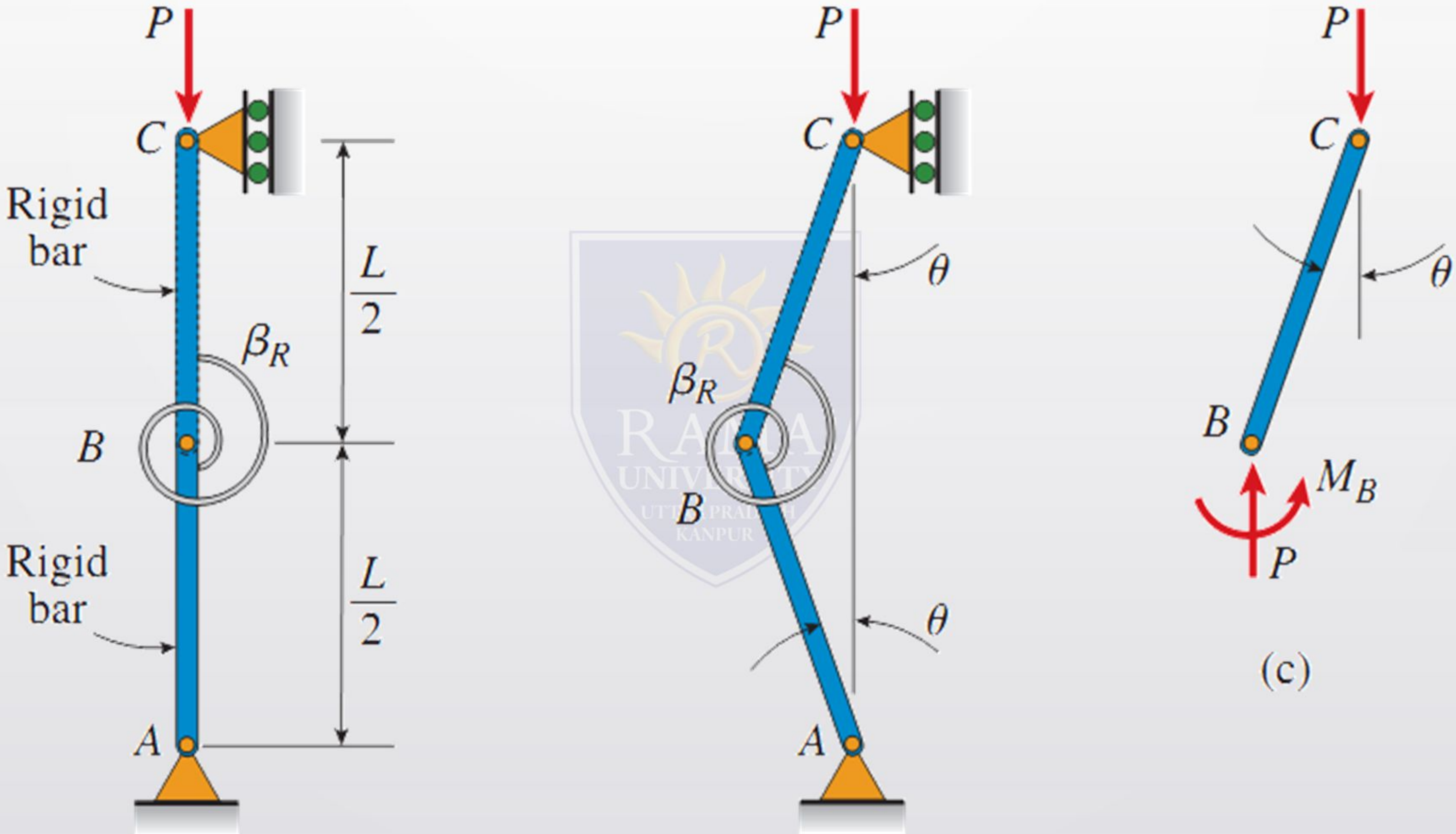
Introduction

- ✓ If a compression member is relatively slender, it may deflect laterally and fail by bending rather than failing by direct compression of the material.
- ✓ When lateral bending occurs, we say that the column has buckled. Under an increasing axial load, the lateral deflections will increase too, and eventually the column will collapse completely.
- ✓ The phenomenon of buckling is not limited to columns. Buckling can occur in many kinds of structures and can take many forms.
  - ✓ When you Step on the top of an empty aluminium can, the thin cylindrical walls buckle under your weight and the can collapses.
  - ✓ When a large bridge, it was found that failure was caused by the buckling of a thin steel plate that wrinkled under compressive stresses.
- ✓ Buckling is one of the major causes of failures in structures, and therefore the possibility of buckling should always be considered in design

Buckling and stability

- ✓ In the idealized structure, the two bars are perfectly aligned and the axial load P has its line of action along the longitudinal axis.
- ✓ Consequently, the spring is initially unstressed and the bars are in direct compression.
- ✓ Now suppose that the structure is disturbed by some external force that causes point B to move a small distance laterally. The rigid bars rotate through small angles θ and a moment develops in the spring. The direction of this moment is such that it tends to return the structure to its original straight position, and therefore it is called a restoring moment. At the same time, however, the tendency of the axial compressive force is to increase the lateral displacement. Thus, these two actions have opposite effects—the **restoring moment** tends to **decrease** the displacement and the **axial force tends to increase** it.

Buckling and stability



(c)

Buckling and stability

- ✓ Now consider what happens when the disturbing force is removed.
- ✓ If the axial force P is relatively small, the action of the restoring moment will predominate over the action of the axial force and the structure will return to its initial straight position.
  - ✓ Under these conditions, the structure is said to be stable.
- ✓ However, if the axial force P is large, the lateral displacement of point B will increase and the bars will rotate through larger and larger angles until the structure collapses.
  - ✓ Under these conditions, the structure is unstable and fails by lateral buckling.

Critical Load

- ✓ The transition between the stable and unstable conditions occurs at a special value of the axial force known as the critical load
- ✓ Taking Moment equilibrium at point B

$$M_B - P\left(\frac{\theta L}{2}\right) = 0$$

$$\left(2\beta_R - P\frac{L}{2}\right)\theta = 0$$

- ✓ From above equation we will get one trivial solution $\theta = 0$ and second solution gives us the critical load

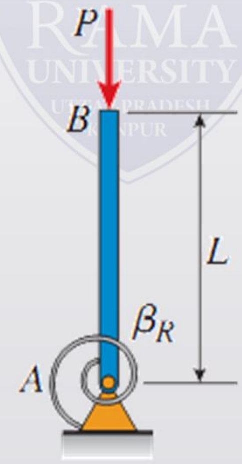
$$P_{cr} = \frac{4\beta_R}{L}$$

- ✓ At the critical value of the load the structure is in equilibrium regardless of the magnitude of the angle θ
  - ✓ **If $P < P_{cr}$ the structure is stable**
  - ✓ **If $P > P_{cr}$ the structure is unstable**
  - ✓ **If $P = P_{cr}$ the structure is neutral equilibrium**
- ✓ At the critical value of the load the structure is in equilibrium regardless of the magnitude of the angle θ
- ✓ The stability of the structure is increased either by increasing its stiffness or by decreasing its length.

Question

11.2-1 The figure shows an idealized structure consisting of one or more **rigid bars** with pinned connections and linearly elastic springs. Rotational stiffness is denoted β_R, and translational stiffness is denoted β.

Determine the critical load P_{cr} for the structure.

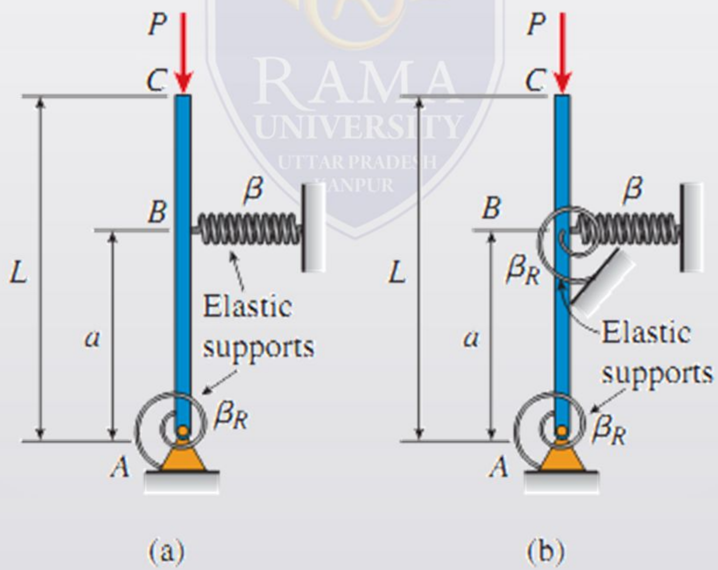


Question

11.2-2 The figure shows an idealized structure consisting of one or more rigid bars with pinned connections and linearly elastic springs. Rotational stiffness is denoted β_R, and translational stiffness is denoted β.

(a) Determine the critical load P_{cr} for the structure from the figure part a.

(b) Find P_{cr} if another rotational spring is added at B from the figure part b.

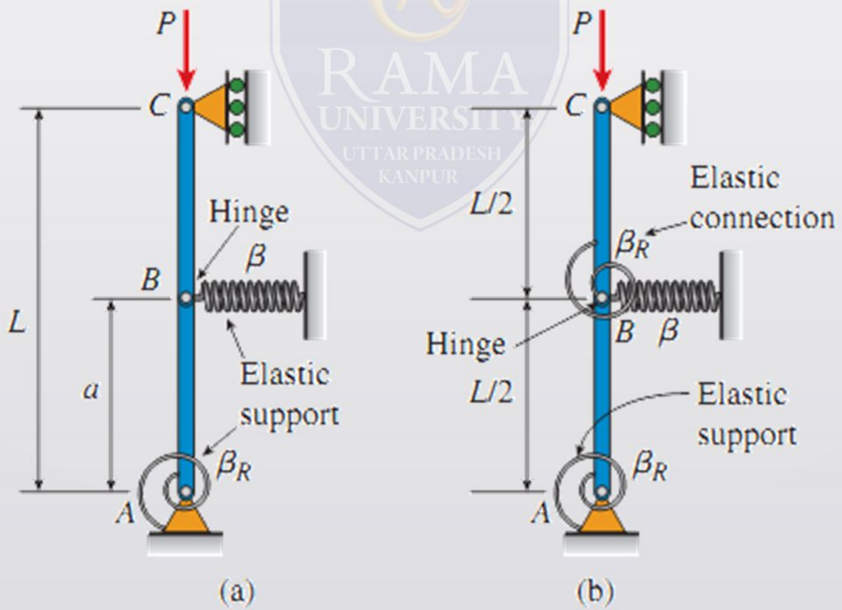


Question

11.2-4 The figure shows an idealized structure consisting of bars AB and BC which are connected using a hinge at B and linearly elastic springs at A and B. Rotational stiffness is denoted β_R and translational stiffness is denoted β.

(a) Determine the critical load P_{cr} for the structure from the figure part a.

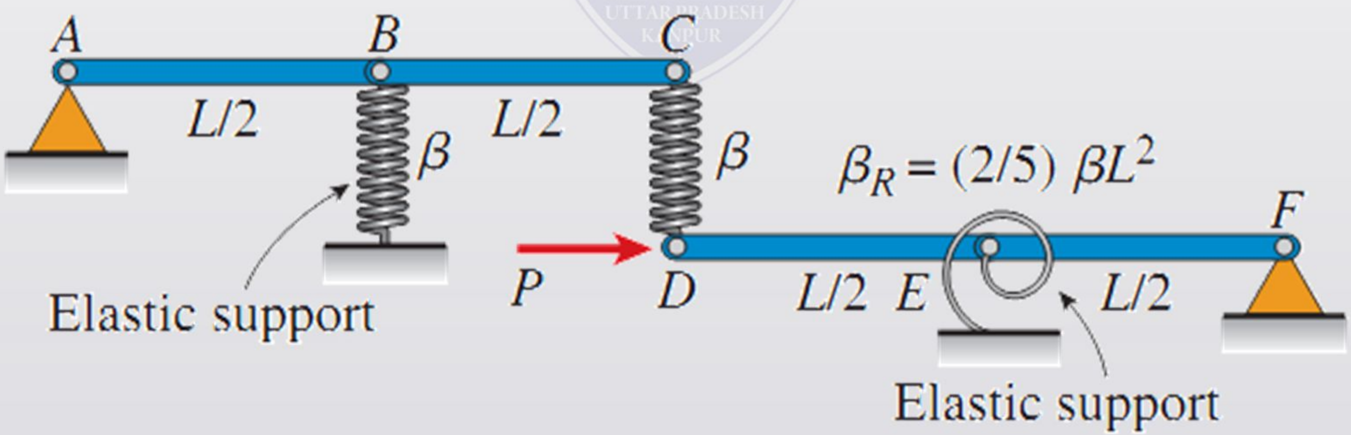
(b) Find P_{cr} if an elastic connection is now used to connect bar segments AB and BC from the figure part b.



Question

11.2-6 The figure shows an idealized structure consisting of rigid bars *ABC* and *DEF* joined by linearly elastic spring β between *C* and *D*. The structure is also supported by translational elastic support β at *B* and rotational elastic support β_R at *E*.

Determine the critical load P_{cr} for the structure.



Question

11.2-7 The figure shows an idealized structure consisting of an L-shaped rigid bar structure supported by linearly elastic springs at *A* and *C*. Rotational stiffness is denoted β_R and translational stiffness is denoted β.

Determine the critical load P_{cr} for the structure.