- A rectangular plate 3 metres long and 1 metre wide is immersed vertically in water in such a way that its 3 metres side is parallel to the water surface and is 1 metre below it. Find: (i) Total pressure on the plate, and (ii) Position of centre of pressure.

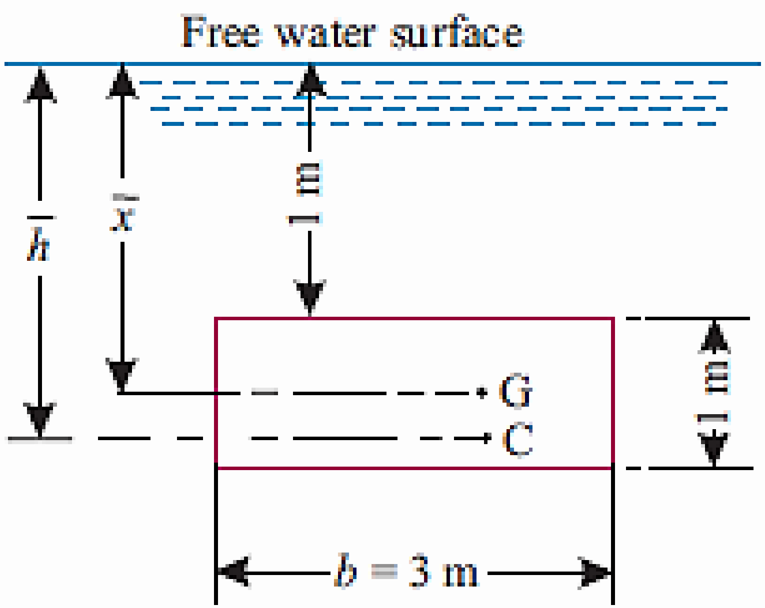
Solution. Width of the plane surface, $b = 3$ m

Depth of the plane surface, $d = 1$ m

Area of the plane surface,

$$A = b \times d = 3 \times 1 = 3 \text{ m}^2$$

$$\bar{x} = 1 + \frac{1}{2} = 1.5 \text{ m}$$



***(i)* Total pressure P:**

Using the relation:

$$P = wA\bar{x} = 9.81 \times 3 \times 1.5$$
$$= \mathbf{44.14 \text{ kN (Ans.)}}$$

***(ii)* Centre of pressure, $\bar{h}$:**

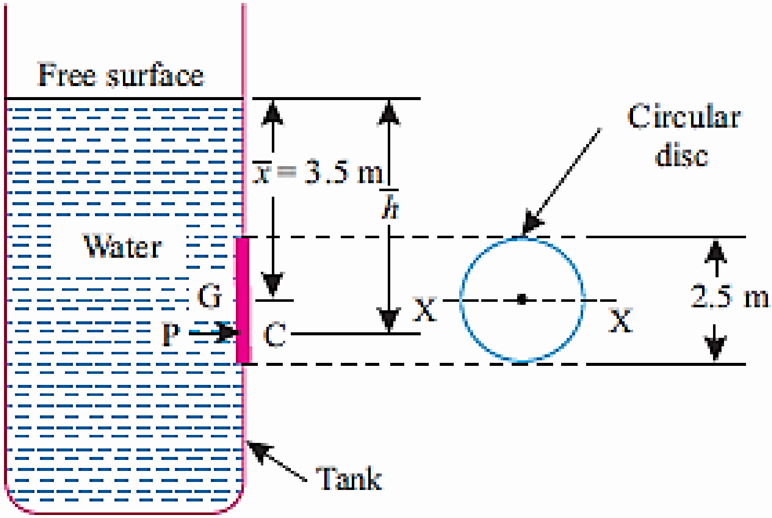
Using the relation: $\bar{h} = \dfrac{I_G}{A\bar{x}} + \bar{x}$

But, $I_G = \dfrac{bd^3}{12} = \dfrac{3 \times 1^3}{12} = 0.25 \text{m}^4$

$\therefore$ $\bar{h} = \dfrac{0.25}{3 \times 1.5} + 1.5 = 1.556$ m

i.e. $\bar{h} = \mathbf{1.556 \text{ m (Ans.)}}$

- A circular opening, 2.5 m diameter, in a vertical side of tank is closed by a disc of 2.5 m diameter which can rotate about a horizontal diameter. Determine: (i) The force on the disc; (ii) The torque required to maintain the disc in equilibrium in vertical position when the head of water above horizontal diameter is 3.5 m.



- Solution. Diameter of the opening, d = 2.5 m

∴ Area of the opening,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 2.5^2 = 4.91 \text{ m}^2$$

Depth of C.G., $\bar{x} = 3.5$ m

(*i*) Force on the disc, P:

Using the relation:

$$P = wA\bar{x} = 9.81 \times 4.91 \times 3.5$$
$$= \mathbf{168.6 \text{ kN (Ans.)}}$$

(*ii*) Torque required, *T*:

In order to determine the torque (*T*) required to maintain the disc in equilibrium, let us first calculate the point of application of force acting on the disc, *i.e.* centre of pressure of the force *P*. The depth of centre of pressure ($\bar{h}$) is given by the relation:

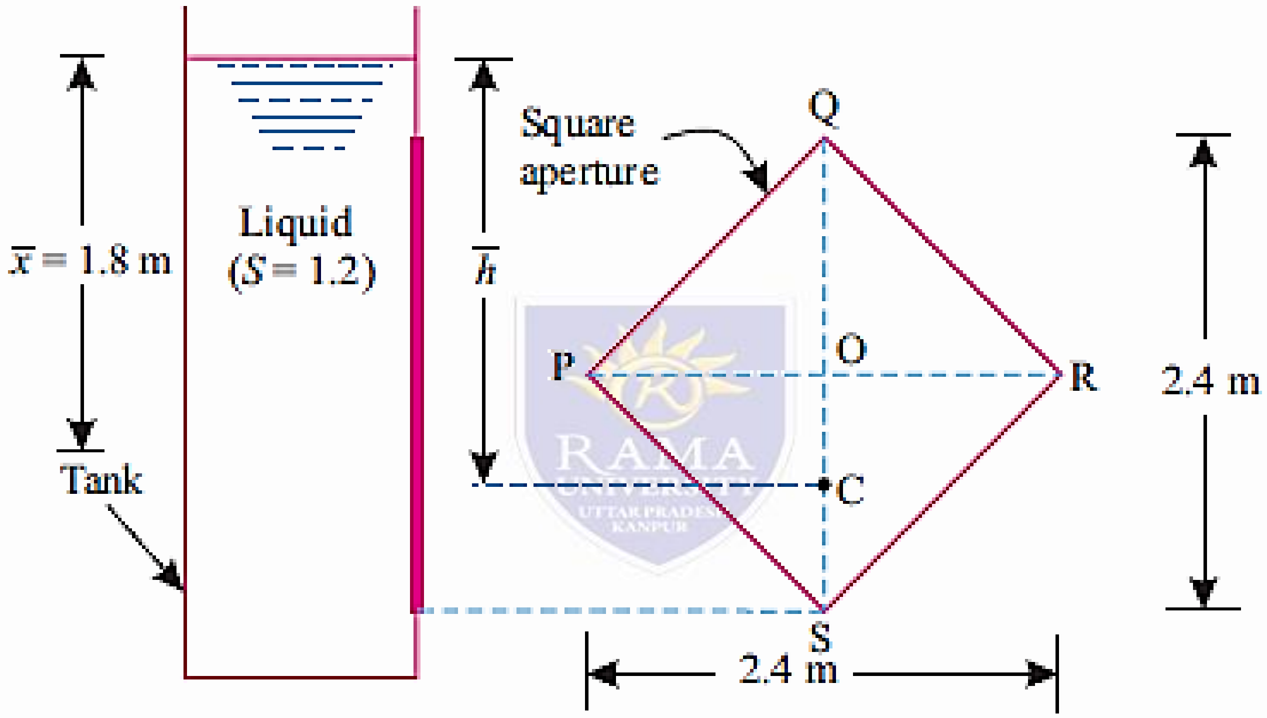
$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(\pi/64 \times d^4)}{(\pi/4 \times d^2)\bar{x}} + \bar{x} \qquad \left[\because I_G = \frac{\pi}{64} \times d^4\right]$$

$$= \frac{(\pi/64 \times 2.5^4)}{(\pi/4 \times 2.5^2) \times 3.5} + 3.5 = 3.61 \text{ m}$$

i.e., the force *P* is acting at a distance of 3.61 m from the free surface. Moment of this force about horizontal diameter *X* – *X*

$$= P(\bar{h} - \bar{x}) = 168.6\,(3.61 - 3.5)$$
$$= 18.55 \text{ kNm.} \qquad \textit{(anticlockwise)}$$

- A square aperture in the vertical side of a tank has one diagonal vertical and is completely covered by a plane plate hinged along one of the upper sides of the aperture. The diagonals of the aperture are 2.4 m long and the tank contains a liquid of specific gravity 1.2. The centre of aperture is 1.8 m below the free surface. Calculate: (i) The thrust exerted on the plate by the liquid; (ii) The position of its centre of pressure.



Solution. Refer to Fig.
Diagonal of aperture, PR = QS = 2.4 m
Area of square aperture, A = area of Δ PQR + area of ΔPSR.

$$= \frac{1}{2} PR \times OQ + \frac{1}{2} PR \times OS$$

$$= \frac{1}{2} \times 2.4 \times \left(\frac{2.4}{2}\right) + \frac{1}{2} \times 2.4 \times \left(\frac{2.4}{2}\right) = 2.88 \text{ m}^2$$

Depth of centre of aperture plate from free liquid surface, $\bar{x} = 1.8$m

(*i*) Thrust exerted on the plate *P*:

Pressure force or thrust on the plate,

$$P = wA\bar{x} = (1.2 \times 9.81) \times 2.88 \times 1.8 = \mathbf{61.026 \text{ kN (Ans.)}}$$

(*ii*) The position of its centre of pressure, $\bar{h}$:

Centre of pressure is given by the relation:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

where, I_G = M. O. I of *PQRS* about diagonal *PR*.

= M.O.I. of Δ *PQR* + M.O.I of *PSR* ...about PR

$$= \frac{2.4 \times (1.2)^3}{12} + \frac{2.4 \times (1.2)^3}{12} = 0.6912 \text{ m}^4 \qquad (\because OQ = OS)$$

[∵ The M.O.I. of a triangle about its base equals $\frac{\text{base} \times (\text{height})^3}{12}$]

$$\therefore \quad \bar{h} = \frac{0.6912}{2.88 \times 1.8} + 1.8 = \mathbf{1.933 \text{ m (Ans.)}}$$

Fluid Properties

- A sliding gate 3 m wide and 1.5 m high lies on a vertical plane and has a coefficient of friction of 0.2 between itself and guides. If the gate weighs 30 kN, find the vertical force required to raise the gate if its upper edge is at a depth of 9 m from free surface of water.

- Solution. Width of the gate, b = 3 m Depth/height of the gate,
- d = 1.5 m Area of the gate, A = b × d = 3 × 1.5 = 4.5 m2 Weight of the gate, W = 30 kN Co-efficient of friction, μ = 0.2

Vertical force required to raise the gate:

Depth of *c.g.* of the gate from water surface,

$$\bar{x} = 9 + \frac{1.5}{2} = 9.75 \text{ m}$$

Pressure force on the gate,

$$P = wA\bar{x} = 9.81 \times 4.5 \times 9.75 = 430.4 \text{ kN}$$

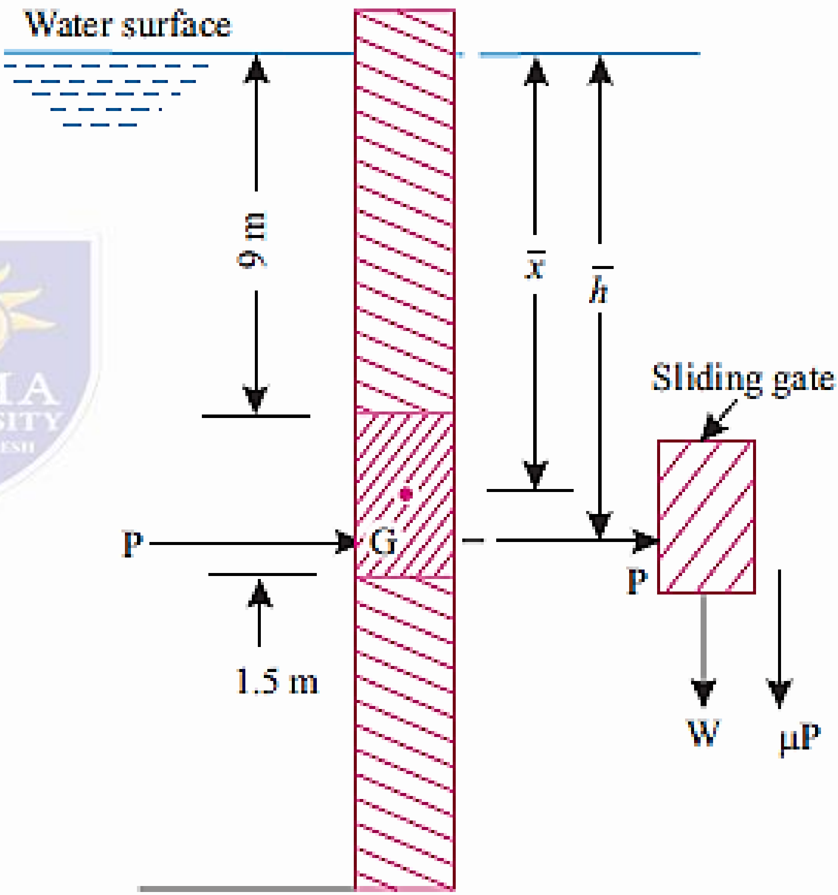
Force required to raise the gate

$$= \text{Frictional force} + \text{weight of the gate}$$

$$= \mu P + W$$

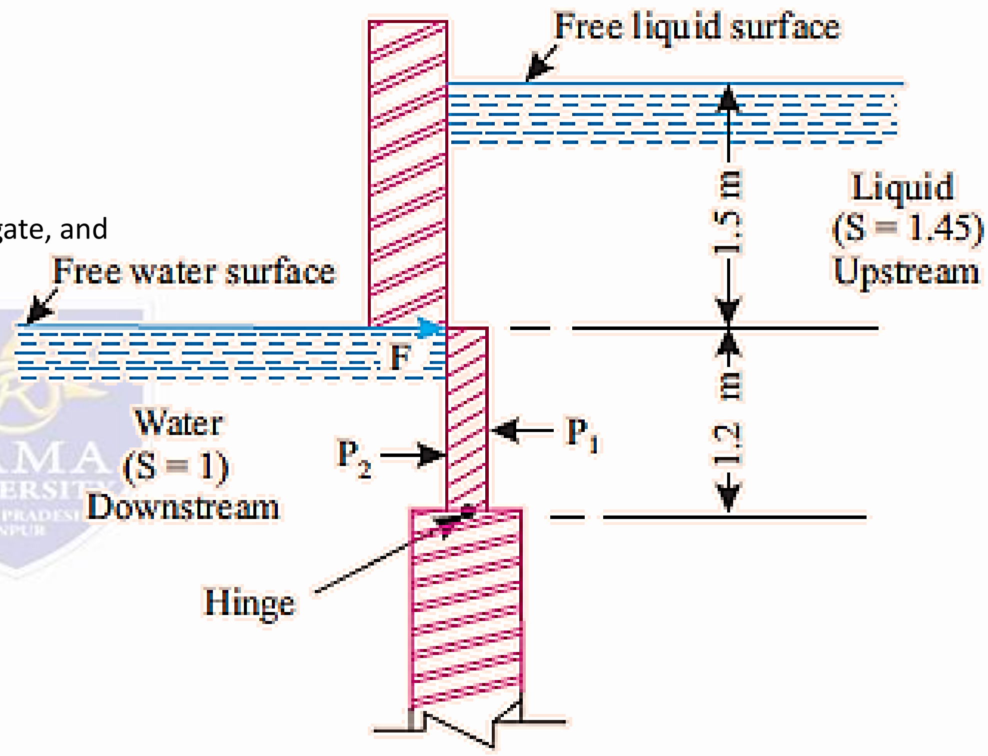
$$= 0.2 \times 430.4 + 30$$

$$= \mathbf{116.08 \text{ kN (Ans.)}}$$



Fluid Properties

- An opening in a dam is covered by the use of a vertical sluice gate. The opening is 2 m wide and 1.2 m high. On the upstream of the gate the liquid of specific gravity 1.45 lies upto a height of 1.5 m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find:(i) The resultant force acting on the gate and position of centre of pressure;
- (ii) The force acting horizontally at the top of the gate which is capable of opening it. Assume that the gate is hinged at the bottom.



- Solution. Width of the gate, b = 2 m
- Depth of the gate, d = 1.2 m
- Area, A = b × d = 2 × 1.2 = 2.4 m2
- Specific gravity of liquid = 1.45
- Let, P1 = Force exerted by the liquid of sp. gravity 1.45 on the gate, and
- P2 = Force exerted by water on the gate.
- (i) Resultant force, P:
- Position of centre of pressure of resultant force:
- We know that, P1 = wA–$x_1$bar
- where, w = 9.81 × 1.45 = 14.22 kN/m3,
- A = 2 × 1.2 = 2.4 m2

$$\bar{x}_1 = 1.5 + \frac{1.2}{2} = 2.1 \text{ m}$$
$$P_1 = 14.22 \times 2.4 \times 2.1 = 71.67 \text{ kN.}$$
$$P_2 = wA\bar{x}_2$$
$$w = 9.81 \text{ kN/m}^3.$$
$$A = 2.4 \text{ m}^2,$$
$$\bar{x}_2 = \frac{1.2}{2} = 0.6 \text{ m}$$
$$P_2 = 9.81 \times 2.4 \times 0.6 = 14.13 \text{ kN.}$$
$$P = P_1 - P_2 = 71.67 - 14.13$$
$$= 57.54 \text{ kN (Ans.)}$$