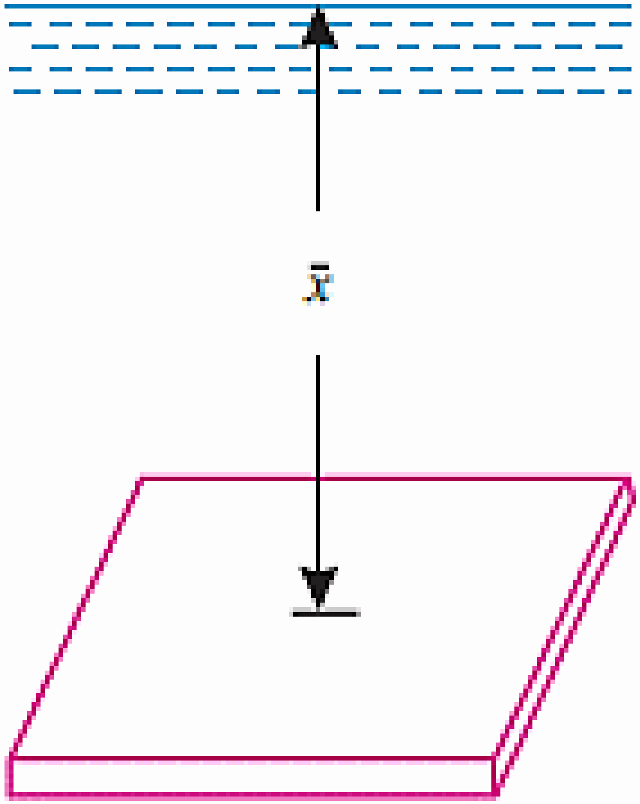
Lecture -19 Hydrostatic Fluid Pressure

- Total pressure. It is defined as the force exerted by static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always at right angle (or normal) to the surface.
- Centre of pressure. It is defined as the point of application of the total pressure on the surface. Now we shall discuss the total pressure exerted by a liquid on the immersed surface. The immersed surfaces may be:
- 1. Horizontal plane surface;
- 2. Vertical plane surface;
- 3. Inclined plane surface;
- 4. Curved surface.

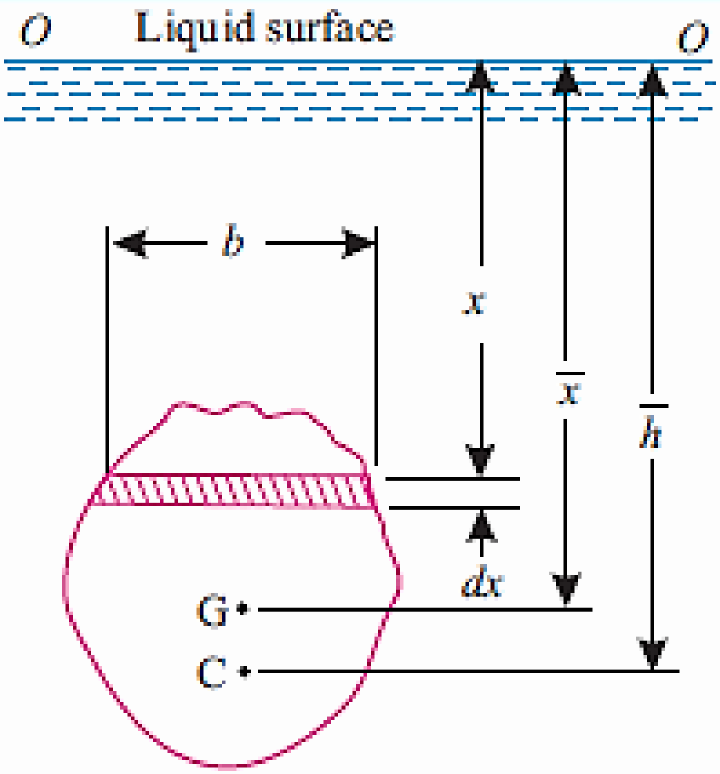
- HORIZONTALLY IMMERSED SURFACE
- Total Pressure (P):
- Refer to Fig. . Consider a plane horizontal surface
- immersed in a liquid.
- Let, A = Area of the immersed surface,
- X (bar) = Depth of horizontal surface from the liquid,
- And w = Specific weight of the liquid.
- The total pressure on the surface,
- P = Weight of the liquid above the immersed surface
- = Specific weight of liquid × volume of liquid
- = Specific weight of liquid × area of surface × depth of liquid

$$= wA\bar{x}$$



Hydrostatic Fluid Pressure

- VERTICALLY IMMERSED SURFACE
- Consider a plane vertical surface of arbitrary shape
- immersed in a liquid as shown in Fig.
- Let, A = Total area of the surface,
- G = Centre of the area of the surface,
- xbar = Depth of centre of area,
- OO = Free surface of liquid, and
- hbar = Distance of centre of pressure from free surface of liquid



- (a) Total pressure (P):
- Consider a thin horizontal strip of the surface of
- thickness dx and breadth b. Let the depth of the strip be x.
- Let the intensity of pressure on strip be p; this may be taken
- as uniform as the strip is extremely small. Then,
- p = wx
- where, w = specific weight of the liquid.
- Total pressure on the strip = p.b.dx. = wx . Bdx

Total pressure on the whole area, $P = \int wx \, . \, bdx = w \int bdx \, . \, x$

But, $\int bdx \, . \, x$ = Moment of the surface area about the liquid level $= A\bar{x}$

$\therefore$ $P = wA\bar{x}$ ...[same as in Art. 3.3]

- or, the total pressure on a surface is equal to the area multiplied by the intensity of pressure at the centre of area of the figure.
- The eqn., P = wAx(bar) holds good for all surfaces whether flat or curved.

- (b) Centre of pressure (h-bar) :
- The intensity of pressure on an immersed surface is not uniform, but increases with depth. As the pressure is greater over the lower portion of the figure, therefore the resultant pressure, on any immersed surface will act at some point, below the centre of gravity of the immersed surface and towards the lower edge of the figure. The point through which this resultant pressure acts is known as ‘centre of pressure’ and is always expressed in terms of depth from the liquid surface.
- Referring to Fig., let C be the centre of pressure of the immersed figure. Then the resultant
- pressure P will act through the point.
- Let, h-bar = Depth of centre of pressure below free liquid surface, and
- I0 = Moment of inertia of the surface about OO.
- Consider the horizontal strip of thickness dx. Total pressure on strip = w.x.b.dx
- Moment of this pressure about free surface OO = (w.x. b.dx) x = w.x^2.b.dx

Total moment of all such pressures for whole area, $M = \int w x^2 . b . dx = w \int x^2 . b . dx$

But, $\int x^2 . b . dx = I_0$ = Moment of inertia of the surface about the free surface OO

econd moment of area)

$$M = wI_0 \quad ...(i)$$

The sum of the moments of the pressure is also equal to $P \times \bar{h}$ ...(ii)

Now equating eqns. (i) and (ii), we get:

$$P \times \bar{h} = wI_0.$$

$$wA\bar{x} \times \bar{h} = wI_0 \quad (\because P = wA\bar{x})$$

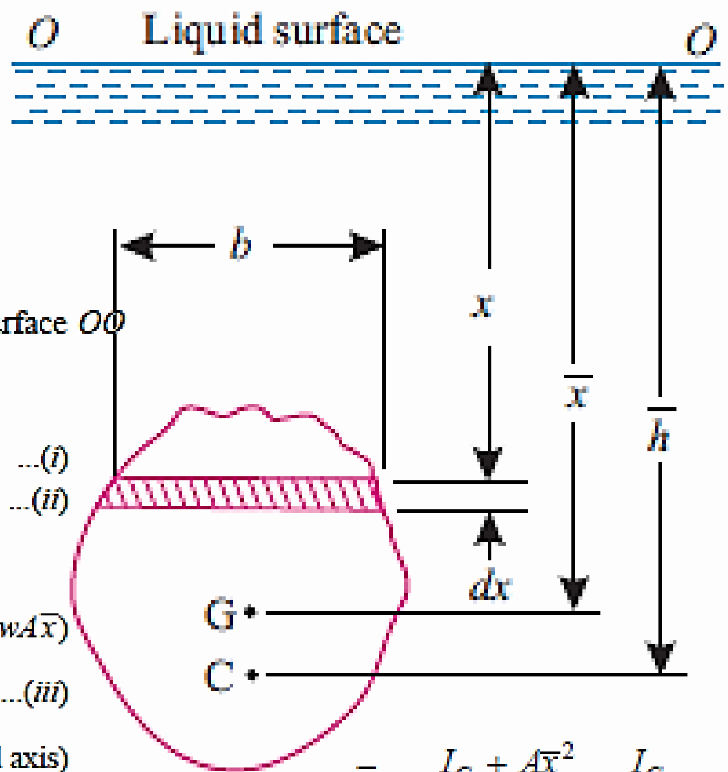
$$\bar{h} = \frac{I_0}{A\bar{x}} \quad ...(iii)$$

Also, $I = I_G + Ah^2$ (Theorem of parallel axis)

where, I_G = Moment of inertia of the figure about horizontal axis through its centre of gravity, and

h = Distance between the free liquid surface and the centre of gravity of the figure ($\bar{x}$ in this case)

Thus rearranging equation (iii), we have

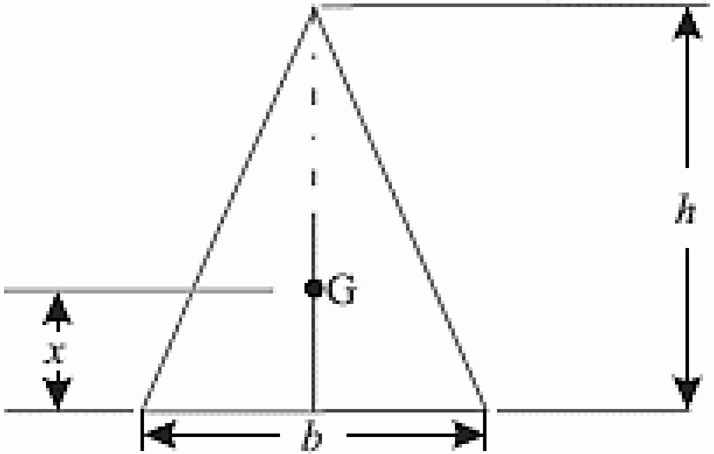


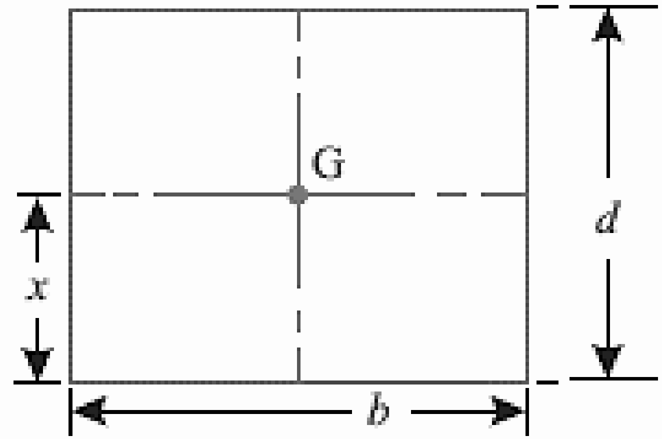
$$\bar{h} = \frac{I_G + A\bar{x}^2}{A\bar{x}} = \frac{I_G}{A\bar{x}} + \bar{x}$$

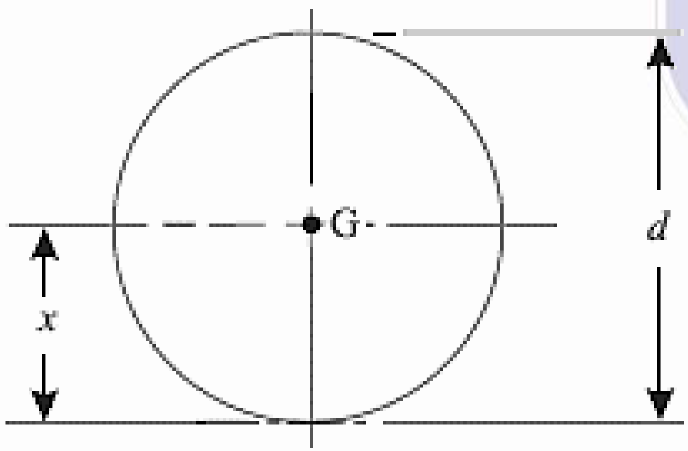
$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

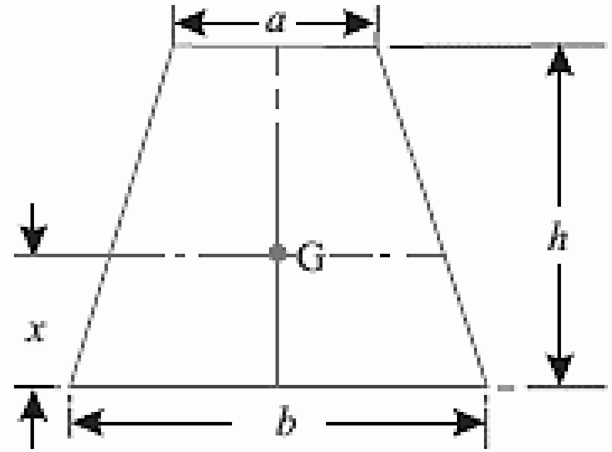
- The Centre of Gravity (G) and Moment of Inertia (I) of Some Important Geometrical Figures:

S.No.	Name of figure	C.G. from the base	Area	I about an axis passing through C.G. and parallel to the base	I about base
1.	Triangle Fig. 3.3	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
2.	Rectangle Fig. 3.4	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
3.	Circle Fig. 3.5	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4.	Trapezium Fig. 3.6	$x = \left[\frac{2a+b}{a+b}\right]\frac{h}{3}$	$\left(\frac{a+b}{2}\right)h$	$\left(\frac{a^2+4ab+b^2}{3b(a+b)}\right) \times h^2$	—


h
G
x
b


G
d
x
b


G
x
d


a
h
G
x
b

Numerical

- Fig. shows a circular plate of diameter 1.2 m placed vertically in water in such a way that the centre of the place is 2.5 m below the free surface of water. Determine: (i) Total pressure on the plate. (ii) Position of centre of pressure.

Solution. Diameter of the plate, $d = 1.2$ m

Area,

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 1.2^2 = 1.13 \text{ m}^2$$

$$\bar{x} = 2.5 \text{ m}$$

(*i*) Total pressure, P:

Using the relation:

$$P = wA\bar{x} = 9.81 \times 1.13 \times 2.5$$

$$= \mathbf{27.7 \text{ kN (Ans.)}}$$

(*ii*) Position of centre of pressure, $\bar{h}$:

Using the relation:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

where,

$$I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times 1.2^4 = 0.1018 \text{ m}^4$$

$$\bar{h} = \frac{0.1018}{1.13 \times 2.5} + 2.5 = 2.536 \text{ m}$$

i.e.

$$\bar{h} = \mathbf{2.536 \text{ m (Ans.)}}$$